Day 18

Kalman Filter

Plant or Process Model

- describes how the system state changes as a function of time, control input, and noise

$$x_{k+1} = \Phi x_k + \Gamma u_k + v_k$$

  - x_k state at time k
  - u_k control inputs at time k
  - v_k process noise at time k
  - Φ state transition model or matrix
  - Γ control-input model or matrix

Measurement Model

- describes how sensor measurements vary as a function of the system state

$$z_k = \Lambda x_k + w_k$$

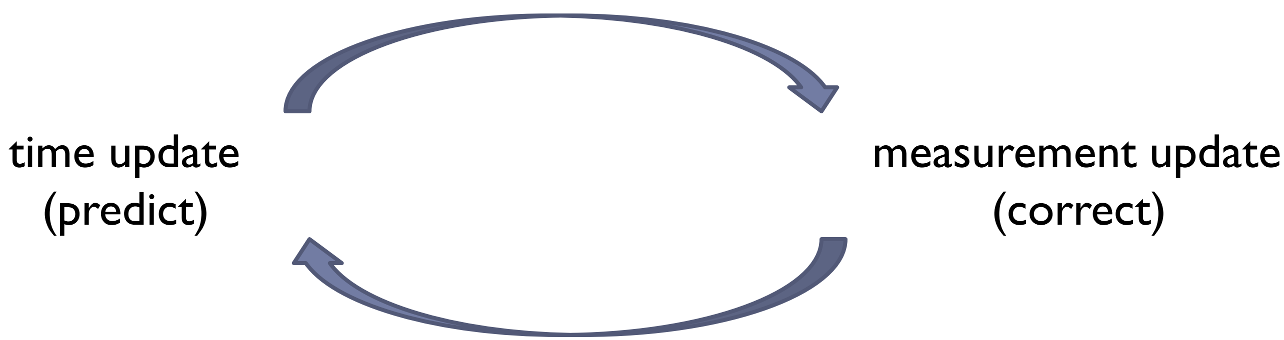
  - z_k sensor measurement at time k
  - w_k sensor noise at time k
  - Λ observation model or matrix

Kalman Filter

- the Kalman filter is a provably optimal (in terms of least-squared error) algorithm for fusing sensor measurements to produce an estimate of the state and the state covariance
  - x_k state at time k
  - P_k state covariance at time k

Kalman Filter

- the Kalman filter estimates a process in two stages
  1. **prediction:** current state and state covariance estimates are projected forward in time to predict the new state and state covariance
     - "time update equations"
  2. **correction:** the sensor measurements are incorporated into the predicted state to obtain improved estimates of the state and state covariance
     - "measurement update equations"



Kalman Filter Algorithm

1. **Initialization**
   - choose (guess) initial values for state and state covariance estimates

$$\hat{x}_0$$

$$P_0$$

Kalman Filter Algorithm

2. **Prediction:**
   - predict the next state using the plant model

$$\hat{x}_{k+1|k} = \Phi \hat{x}_k + \Gamma u_k$$

   - state covariance grows (because we are not incorporating the sensor measurements yet)

$$P_{k+1|k} = \Phi P_k \Phi^T + C_{v_k}$$

     - C_{v_k} covariance of the plant noise

Kalman Filter Algorithm

3. **Correction:** correct the predicted state using the sensor measurement
   - expected value of measurements (from measurement model)

$$\hat{z}_{k+1} = \Lambda\, \hat{x}_{k+1|k}$$

   - difference between actual and expected measurements

$$r_{k+1} = z_{k+1} - \hat{z}_{k+1}$$

   - measurement covariance

$$S_{k+1} = \Lambda\, P_{k+1|k}\, \Lambda^T + C_{w_{k+1}}$$

   - Kalman gain

$$K_{k+1} = P_{k+1|k}\, \Lambda^T\, S_{k+1}^{-1}$$

Kalman Filter Algorithm

4. **State and state covariance:**

- new state estimate incorporating most recent measurement

$$\hat{x}_{k+1} = \hat{x}_{k+1|k} + K_{k+1}\, r_{k+1}$$

- new state covariance estimate

$$P_{k+1} = \left(I - K_{k+1}\, \Lambda\right) P_{k+1|k}$$